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NonQCD contributions to heavy quark masses and sensitivity to Higgs mass

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Abstract

We find that if the Higgs mass is close to its present experimental lower limit (100 GeV),Yukawa interactions in the quark-Higgs sector can make substantial contribution to the heavy quark MS masses.

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1 Introduction

Recent reports from ALEPH,DELPHI, and SLD [1] of measurements of the $\overline{MS}$ mass of the b-quark at the mass scale of the Z mass, combined with previous determinations at the scale of the b mass , show the running of this mass, and invite comparison with theory. So far, this has been done entirely in the context of QCD. In this paper we note that because the t-quark is so heavy its Yukawa coupling to the scalar sector (including the charged Goldstone bosons in Feynman-t'Hooft gauge) is comparable to the strong QCD coupling constant at the scale of M_Z. Here we consider what role these nonQCD interactions may play in the analysis of these observations.

II One-loop perturbative treatment

We work in a truncated version of the standard model in which only the strong QCD gauge coupling (g_3 or g_s),the Yukawa couplings of the heavy t and b quarks, and the quartic self coupling of the scalar fields are kept nonzero. The weak gauge couplings (g_1, g_2) and all the other Yukawa couplings are taken to zero.(and $m_b \ll m_t$ is set to zero except when it is the object being determined). For physical values of these masses and coupling constants, these approximations are too crude for precision calculations, but they provide dramatic simplification and hence are very useful for orientation for more heavy duty calculations, and in some cases, are sufficiently dominant to provide useful estimates.

We compute the one-point and two-point functions of the model through one-loop order in $\overline{MS}$ renormalization and in on-shell momentum subtraction renormalization (OS MOM) in order to determine the counter terms required for these two different renormalization schemes. The relation between the perturbative pole mass and the $\overline{MS}$ mass follows from the pole condition

$$0 \;=\; m^\star - \bar{m} - \overline{\Sigma}(m^\star) \tag{1}$$

We use m for generic renormalized mass,$\bar{m}$ for $\overline{MS}$ masses, and (sometimes) $m^\star$ for perturbative pole mass.

(1) t-quark $\overline{MS}$ mass

Through two loop order for QCD [2] and one loop for electroweak [3][4] , the relation for t-quark is

$$\bar{m}_t(\mu) = m_t^\star\{1 - \frac{\alpha_s}{\pi}(\frac{4}{3} + \ln\frac{\mu^2}{m_t^2})$$
$$-(\frac{\alpha_s}{\pi})^2((15.37 - 1.04\, n_f) + (\frac{7}{6} - \frac{185}{24} + \frac{13}{36}n_f)\ln\frac{\mu^2}{m_t^2} + (\frac{1}{2} - \frac{11}{8} + \frac{1}{12}n_f)(\ln\frac{\mu^2}{m_t^2})^2)$$

$$+\frac{y_t^2}{16\,\pi^2}(-\frac{1}{2}d(r)+\frac{3}{4}\ln\frac{\mu^2}{m_t^2}+\frac{2N_c}{r^2}(1+\ln\frac{\mu^2}{m_t^2})-\frac{\lambda}{16\,\pi^2}(3+3\ln\frac{\mu^2}{M_h^2})\} \quad (2)$$

Here $N_c = 3$ and

$$r=\frac{M_h}{m_t}, \qquad d(r)=-\frac{1}{2}+\int_0^1 dx(2-x)\ln(r^2(1-x)+x^2) \quad (3)$$

For a perturbative treatment, we input to the right hand side the star quantities which are either measurable or expressible as combinations of measurable quantities (We consider that the pole mass of a heavy quark can be determined up to an ambiguity of order Λ_{QCD})

$$m_t^\star = 174 \qquad v^\star = 250 \qquad m_b^\star = 4.9 \quad (4)$$

The Higgs mass is put in as one of a series of values ranging from 100 to 300 GeV. The coupling constants are fixed by the relations

$$y_i=\sqrt{2}\frac{m_i}{v}, \quad i=t,b, \qquad \lambda=\frac{m_h^2}{2v^2} \quad (5)$$

In particular, $y_t^\star = 0.9843$ while $y_b^\star$ is so small that we neglect its contribution to other quantities. These relations hold for all orders for bar and star quantities.

The conventional definition of *the* $\overline{MS}$ mass is $\bar{m}_t(\bar{m}_t)$. With this definition, the $\ln(\frac{\mu^2}{m_t^2})$'s in (2) are zero, and (2) reduces to

$$\begin{aligned}\bar{m}_t &= m_t^\star\{1-\frac{\alpha_s}{\pi}(\frac{4}{3})-(\frac{\alpha_s}{\pi})^2(15.37-1.04\,n_f)\\ &+\frac{{y_t^\star}^2}{16\,\pi^2}(-\frac{1}{2}d(r)+\frac{2N_c}{r^2})-\frac{\lambda^\star}{16\,\pi^2}(3+3\ln\frac{m_t^2}{M_h^2})\end{aligned} \quad (6)$$

In Table 1. we give values of the nominal $\overline{MS}$ top quark mass,$\bar{m}_t(\bar{m}_t)$, computed with just the QCD contribution (α_s) and with the nonQCD contribution (y_t, λ) included. We see sensitivity to the Higgs mass. For the lightest Higgs considered, $m_h = 100$, the nonQCD contribution is twice as large as and of opposite sign as the QCD contribution. For the heaviest Higgs,$m_h = 300$, the nonQCD contribution is only a small correction to the QCD only result.

(2) b-quark $\overline{MS}$ mass

For the b-quark, the flavor independent QCD contribution to the relation is the same, but the electroweak contribution is different.

$$\bar{m}_b(\mu) = m_b^{\star}\{1 - \frac{\alpha_s}{\pi}(\frac{4}{3} + \ln\frac{\mu^2}{m_b^2})$$
$$-(\frac{\alpha_s}{\pi})^2((15.37 - 1.04\, n_f) + (\frac{7}{6} - \frac{185}{24} + \frac{13}{36} n_f)\ln\frac{\mu^2}{m_b^2} + (\frac{1}{2} - \frac{11}{8} + \frac{1}{12} n_f)(\ln\frac{\mu^2}{m_b^2})^2)$$
$$+\frac{y_t^2}{16\,\pi^2}(-\frac{5}{8} - \frac{3}{4}\ln(\frac{\mu^2}{m_t^2}) + \frac{2N_c}{r^2}(1 + \ln(\frac{\mu^2}{m_t^2}))) - \frac{\lambda}{16\,\pi^2}(3 + 3\ln\frac{\mu^2}{M_h^2})\} \quad (7)$$

Now when we set $\mu = m_b$ to get the nominal $\overline{MS}$ b-quark mass, the QCD logs are again zero but in the electroweak contribution there remain large logarithms of $\frac{m_b}{m_t}$. These logs are not the result of running anything from $\mu = m_t$ to $\mu = m_b$. They arise from two features of the electroweak interactions which are not present in QCD. First is the existence of large mass splitting between members of a multiplet, in particular, t and b. The first $ln\frac{m_t}{m_b}$ in (7)comes from the b-quark self-energy diagram in which a b-quark emits a virtual charged Goldstone boson and propagates as a t-quark until the boson is reabsorbed. In QCD gluons change color, not flavor, and quarks of same flavor, but different color have same mass. Second is the feature of Spontaneous Symmetry Breaking for mass generation and the concurrent existence of a scalar field which acquires a vev. When the scalar field is shifted to get into the proper vacuum, the vev propagates into the boson 1-,2-,and 3-point functions and the fermion 2-point functions, and is the source of the second $\ln\frac{m_t}{m_b}$ and the $\ln\frac{M_h}{m_b}$ in (7). (For more detail, see the appendix).

For the smaller Higgs masses considered (100,150 GeV), the perturbative mass shifts with μ fixed at 4.9 are large enough that the condition $\bar{m} = \bar{m}(\mu = \bar{m})$ is badly violated, so we let μ float and solve by iteration until the μ put into (7) is the same as the output $\bar{m}(\mu)$. The results are given in Table.2. For Higgs mass of 300 GeV, the electroweak contribution is again a small correction to QCD contribution, but for lighter Higgs, including the electroweak contribution leads to substantially smaller $\overline{MS}$ b-quark mass than given by QCD contribution alone. For Higgs mass 100 GeV, the iteration is unstable and does not converge to any positive value. (The connection between a "light" Higgs and breakdown of $\overline{MS}$ perturbation theory has been noted before [11]).

III Evolution of $\bar{m}_b(\mu)$ from μ equal m_b to M_Z

From $\mu = 4.9 \;\; to \;\; 91.19$ is a large change in scale, leading to large $\ln[\frac{M_Z^2}{m_b^2}$. We therefor differentiate (7) to obtain the one-loop differential equation for the running of $\bar{m}_b(\mu)$ By keeping just the dominant one-loop QCD and electroweak Yukawa terms,

we can obtain a simple analytic solution. With $t = \ln(\frac{\mu}{4.9})$

$$\frac{d}{dt}\bar{m}_b(t) = m_b^\star\{\frac{g_s^2(t)}{16\pi^2}(-8) + \frac{y_t^2(t)}{16\pi^2}(\frac{4N_c}{r^2}\} \equiv -m_b^\star A(t) \tag{8}$$

$$\bar{m}_b(t) = m_b^\star \exp(-\int_{t_0}^t dt' A(t')) \quad + const \tag{9}$$

The initial condition

$$\bar{m}(t_0) = \bar{m}$$

fixes

$$const = \bar{m} - m^\star$$

Then we need the solutions of the one-loop RG equations for the QCD and electroweak Yukawa coupling constants. The one-loop QCD coupling constant is

$$g_s^2 = \frac{g_{s_0}^2}{1 + g_{s_0}^2 b_0 t} \tag{10}$$

with

$$b_0 = \frac{1}{16\pi^2}(22 - \frac{4}{3}n_f)$$

and $g_{s_0}^2 = 2.599$ fixed to make $\alpha_s = .119$ at $\mu = M_Z$.

For the electroweak Yukawa coupling constant, we start with the equation relating $\bar{y}$ and $y^\star$ [7]

$$\bar{y}_t^2 = {y_t^\star}^2\{1 - \frac{\alpha_s}{\pi}(\frac{8}{3} - 2\ln\frac{m_t^2}{\mu^2}) + \frac{y_t^2}{16\pi^2}(-(N_c + \frac{3}{2})\ln\frac{m_t^2}{\mu^2} + \frac{N_c}{2} - d(r)) + \frac{\lambda}{16\pi^2}(1) \tag{11}$$

Differentiating, the one-loop RG equation for $\bar{y}_t^2$ is ($y = y_t = y_{top}$ to be distinguished from $t = \ln\frac{\mu^2}{4.9}$)

$$\frac{d}{dt}y^2 = \frac{1}{16\pi^2}((2N_c + 3)(y^{2^2} - 12C_F g_s^2 y^2) \tag{12}$$

After substitution of $g_s^2(t)$ (10)into (12) the resulting differential equation for $y^2(t)$ can be solved.

$$y^2(t) = y_0^2\{(1+\gamma t)^{\frac{b}{\gamma}} + \frac{a y_0^2}{\gamma - b}(1+\gamma t)^{\frac{b}{\gamma}} - (1+\gamma t))\}^{-1} \tag{13}$$

with

$$a = \frac{9}{16\pi^2}, \qquad b = \frac{g_{s_0}^2}{\pi^2} \qquad \gamma = b_0 g_{s_0}^2$$

and the initial condition that $\bar{y}_t^2(\mu = m_t)$ is given by (11) fixes y_0^2. With $\bar{g}_s^2(t), \bar{y}_t^2(t)$ determined, the integral (9) can be done numerically ($t_Z = \ln \frac{M_Z}{4.9} = 2.9237$).

$$\int_{t_0}^{t_Z} dt' A(t') \equiv F$$

and

$$\bar{m}_t(t_Z) = m_b^\star e^{-F} + \bar{m}_b - m_b^\star \tag{14}$$

Results are given in Table.3.

There is no entry in the table for $M_h = 100$ GeV because there is no solution for that M_h in table.2. The values of $\bar{m}_b(t_Z)$ obtained for M_h ranging from 150 to 250 GeV fairly well span the range of experimental values [1] For $M_h = 300$ GeV the obtained value is probably high. Global fits to electroweak precision data [6] disfavor Higgs mass greater than 200 GeV. For Higgs mass from 150 to 200 GeV, the values of $\bar{m}_b(M_Z)$ in Table.3 are consistent with the data while the values of $\bar{m}_b(\bar{m}_b)$ in Table.2 are not consistent with the generally accepted value (4.1 to 4.3 GeV). This leads us to question the provenance of the generally accepted value of $\bar{m}_b(\bar{m}_b)$. It comes from QCD sum rule calculations, Lattice gauge calculations, and potential models. All done in context of pure QCD. Thus it is necessary to investigate the contributions of the electroweak Yukawa couplings to these calculations and see if they might then agree with the smaller values we have obtained for $\bar{m}_b(\bar{m}_b)$ from the transformation (7), in the case of the smaller Higgs masses.

The calculations just referred to are generally substantial undertakings without the added complication of the electroweak Yukawa contributions, and we do not propose to repeat and generalize any of them in this paper. However, we can identify one dominant term responsible for most of the electroweak contribution seen in Tables 2 and 3, for the lighter Higgs masses. And to incorporate that term into a simple version of the QCD sum rule analysis is not a major undertaking.

IV Electroweak contribution to QCD sum rule analysis.

Returning to (7) we see that for light Higgs the term $\frac{y_t^2}{16\pi^2}\frac{2N_c}{r^2}$ is almost the whole electroweak contribution. This term appears as an $\overline{MS}$ counter term for the quark selfenergy function. Its origin is in the elimination of the tadpoles introduced by the shift of the Higgs field to do perturbation theory about the stable vacuum. See Appendix A and [7]. In the QCD sum rule calculations, one computes moments of the dispersion representation of the vacuum polarization.

$$i\int dx e^{iqx} < 0|T(j_\mu(x)j_\nu(0))|0 >= (q_\mu q_\nu - q^2 g_{\mu\nu})(C_I(q^2) + \dots \tag{15}$$

$$C(Q^2) = -\frac{Q^2}{\pi}\int_{4m^2}^{\infty} ds \frac{\Im C(s)}{s(s+Q^2)} \tag{16}$$

$$\mathcal{M}_n = \frac{1}{\pi}\int ds \frac{\Im C(s)}{s^{n+1}} = \frac{1}{12\pi^2 Q_q^2}\int ds \frac{R(s)}{s^{n+1}} \tag{17}$$

The QCD Feynman diagrams we include are the basic one-and two-loop vacuum polarization diagrams (contributions to the moments available in the literature [8]) and two diagrams with a single $\frac{\delta m}{m}$ mass insertion in one or the other of the quark propagator lines in the first diagram . The new contribution to $\Im C(s)$ is

$$\Im\delta C(s) = -\frac{2N_c}{\pi}\frac{\delta m}{m}\frac{m^4}{s^2}\frac{1}{\sqrt{1-\frac{4m^2}{s}}} \tag{18}$$

with

$$\frac{\delta m}{m} \simeq -\frac{y_t^2}{16\pi^2}\frac{2N_c}{r^2}(1-\ln\frac{m_t^2}{m_b^2}) \tag{19}$$

This gives calculated

$$\mathcal{M}_n = \mathcal{M}_n^{(0)} + \mathcal{M}_n^{(1)} + \delta\mathcal{M}_n^{(1)} \tag{20}$$

to be matched against the integral of R(s) evaluated from experimental data on the upsilon resonance plus continuum.

In the purely QCD analysis, the convergence of the perturbation series is acceptable up to fairly high n moments.

Recently Jamin and Pich [9] have done an exhaustive treatment of the purely QCD analysis. They found that the optimal choice of moments to employ was n from 7 to 15. (Smaller n is sensitive to poorly known continuum threshold, and for larger n perturbation theory gets worse and nonperturbative contributions come into play) Furthermore the value of the mass obtained is rather stable in this range. But precisely because the dominant $\overline{MS}$ electroweak term is so large,the perturbative treatment fails rapidly as the order n of the moment increases. We give the results for the moment $\mathcal{M}_1$ in table.4. For the integral over R in (17)we used PDG data for six upsilon resonances and asymptotic form 0f R for continuum starting at 11.1 GeV. There is no entry in Table.4 for Higgs mass 100 GeV. For this value of Higgs mass, the perturbative calculation gives an unphysical negative value for $\mathcal{M}_1$ So again we see the breakdown of the $\overline{MS}$ electroweak perturbation series for this "light" Higgs. For the other values of M_h from 150 to 300 GeV, the values of $\bar{m}_b(\bar{m}_b)$ obtained from this crude implementation of the QCD sum rule, augmented by electroweak contribution, look at least recognizably like the results in Table.2,obtained from the transformation equation (7).

V.Conclusions

Although none of the numerical results presented here are definitive - the approximations have been too many and too crude, a consistent pattern has emerged, with nonnegligible electroweak contributions and values of $\bar{m}_b(\bar{m}_b)$ substantially smaller than currently accepted, and furthermore, sensitive to the mass of the Higgs boson. We have started from the position that we have a pretty good knowledge of the perturbative pole mass of a heavy quark from threshold behavior,potential models,and lattice calculations. We have chosen 4.9 GeV (and don't protest if someone prefers 4.8 or 5.0). We have then used the perturbative definition of the pole mass and $\overline{MS}$ perturbation theory to convert to the nominal $\bar{m}_b$ mass $\bar{m}_b(\bar{m}_b)$. We pointed out some features present in the electroweak sector which do not appear in QCD. And we found values of $\bar{m}_b$ which depend on the Higgs mass, and for range of Higgs masses favored by global fits to precision electroweak data, are significantly smaller than currently accepted value. At this point in the argument we are in conflict with one or the other of two accepted positions: M_h should be less than 200 GeV and $\bar{m}_b$ is 4.1 to 4.3. We pointed out that this value for $\bar{m}_b$ is not directly a measured number, but depends on the analysis used to fit experimental data. In particular, the analyses which led to this value are all purely QCD. We then made an (admittedly crude) analysis of what value might come out of the QCD sum rule approach if one included the electroweak contribution. The results were gratifyingly similar to those obtained from the conversion from pole mass. We also carried out the evolution 0f $\bar{m}_b$ from $\mu = \bar{m}_b$ to $\mu = M_Z$, and found results consistent with the recently reported experimental results. There is a possible problem here. Just as in the case of $\bar{m}_b(\bar{m}_b)$, the analysis presented in the experimental papers was purely $\overline{MS}$ QCD. I have not found a quick way to incorporate the electroweak contribution in this analysis, but I refer to a paper by Rodrigo [10] which is aware of the possibility of important electroweak Yukawa contributions, but claims that they can be suppressed by taking appropriate ratios of ratios of the experimental data. Then our analysis of the evolution including electroweak contribution would remain consistent with the experimental results. Finally, the dominant large electroweak contribution we have exploited,$\delta\bar{\zeta}_v$, is specific to the $\overline{MS}$ scheme. As explained in the Appendix, in a theory with a global symmetry and Goldstone theorem,$\delta\zeta_v^\star$ may simply be taken to be zero in a MOM scheme. There will still be a strong y_t Yukawa coupling connecting the two members of the t,b-doublet,which suggests that even with $\delta\zeta_v^\star = 0$,the electroweak Yukawa interactions may make a contribution to the QCD sum rule analyses. From the point of view of a MOM scheme, the various $\bar{m}_b$'s are a nonissue, and the QCD sum rules become

a more general testing ground for the whole theory, QCD plus electroweak . In the absence of one dominant contribution, it will be necessary to do a complete perturbative calculation of the electroweak contribution to the QCD moment functions, to see how it works out.

A details of renormalization schemes

Since the $2N_c y^2(\frac{m_t}{M_h^2})$ term is the primary source of our results which differ from the received wisdom,we explain here in some detail how it arises and is propagated in the $\overline{MS}$ two-point and three-point functions

The original Lagrangian for the Higgs-Goldstone sector is

$$\mathcal{L} = (\partial\Phi_B^\dagger)(\partial\Phi_B) - \mu_B^2\Phi_B^\dagger\Phi_B - \lambda_B(\Phi_B^\dagger\Phi_B)^2 \tag{21}$$

with

$$\Phi_B = \begin{pmatrix} \Phi_+^B \\ \Phi_0^B \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\phi_1^B - i\phi_2^B) \\ \frac{1}{\sqrt{2}}(h^B - \imath\phi_3^B) \end{pmatrix} \tag{22}$$

For $\mu_0^2 < 0$ we are in the broken symmetry phase and $< h_B >$ is not zero in the stable vacuum. In order to be perturbing about the true ground state, rewrite everything in terms of the shifted field which has zero vev in the true ground state.

$$< h_B >= V_B, \qquad h_B = V_B + \hat{h}_B, \qquad < \hat{h}_B >= 0 \tag{23}$$

Renormalize

$$\begin{aligned} \Phi_B &= \sqrt{Z_\phi}\Phi \\ V_B &= \sqrt{Z_\phi}V \\ \mu_B^2 = Z_{\mu^2}\mu^2, &\qquad \lambda_B = Z_\lambda\lambda \end{aligned} \tag{24}$$

The quartic scalar vertex renormalization is $Z_4 = Z_\lambda Z_\phi^2$

The complete all orders vev V is determined by the requirement that the vev of $\hat{h}$ be zero, order by order in the loop expansion. Our notation for the order by order expansion of V is

$$\begin{aligned} V &= \zeta_v v \\ \zeta_v &= 1 + \frac{\delta V}{v} \quad = 1 + (\zeta_v - 1) \quad = 1 + \delta\zeta_v \end{aligned} \tag{25}$$

We use the notation ζ_v rather than Z_v, to emphasize that (25) is not the renormalization of any bare field or bare parameter in the Lagrangian (21). It is the loop expansion of the vev of the renormalized field (and is UV finite in the truncated

theory considered here, but both UV divergent and gauge variant when the gauge couplings are turned on).

The field shift (23) generates in the Lagrangian (21) terms linear in the shifted field.

$$\mathcal{L}_{linear} = -V(Z_\phi Z_{\mu^2}\mu^2 + Z_4\lambda V^2)\hat{h}$$
$$= -v(\mu^2+\lambda v^2)\hat{h} - [((\zeta_v - 1)(\mu^2+\lambda v^2)) - v((Z_\phi Z_{\mu^2}-1)\mu^2 + (Z_4\zeta_v^2 - 1)\lambda v^2)]\,\hat{h} \tag{26}$$

Treated as a perturbing interaction, the first term in (26) will generate tree tadpole diagrams. We eliminate them by fixing the tree level vev of the unshifted field as

$$v^2 = \frac{-\mu^2}{\lambda}, \qquad\qquad (\mu^2 < 0) \tag{27}$$

Then inspection of the terms in the Lagrangian which are quadratic in the fields after the shift, leads to identification of the masses,

$$M_\phi^2 = 0, \qquad\qquad M_h^2 \equiv M^2 = 2\lambda v^2 \tag{28}$$

Although this is a relation between quantities identified at tree level,in our treatment there are no higher order corrections. In an OS MOM scheme this mass is the physical Higgs mass (modulo problems with unstable particles) ([11]).

The field shift also generates in (21) terms trilinear in the scalar fields. There are also trilinear Yukawa couplings between the quarks and the scalars, which have been the subject of the bulk of this paper. These terms generate one-loop tadpole diagrams (fig.1). After the condition (27) has been applied, (26) is still a term of the Lagrangian linear in the shifted field, but the coefficients are formally of one-loop order, so the tadpole trees generated by it now act as counter terms to the one-loop tadpole diagrams (fig.1).

$$\mathcal{L}^{(1)}_{linear} = v(Z_\phi Z_{\mu^2} - Z_4\zeta_v^2)\frac{M^2}{2}\hat{h} \equiv v\delta\mu^2\hat{h} \tag{29}$$

The elimination of the one-loop tadpoles fixes this particular combination of $\delta Z's$ and $\delta\zeta_v$

$$\delta\mu^2 = 3\lambda(A_0 + A_M) - 2N_c y^2 A_m, \qquad\qquad (m = m_t) \tag{30}$$

A is the standard tadpole loop integral, dimensionally regularized.

$$A_M = i\int_{reg}(d\ell)\frac{1}{\ell^2 - M^2 + \imath\delta} = \frac{M^2}{16\pi^2}(-\Delta_\epsilon + \ln\frac{M^2}{\mu^2} - 1) \tag{31}$$

$$\Delta_\epsilon = \frac{2}{\epsilon} - \gamma_E + \ln 4\pi, \qquad\qquad \epsilon = 4 - d$$

In $\overline{MS}$ all of the $\delta Z's$ are only of form constant times Δ_ϵ and cannot cancel log and constant pieces of combined (30),(31). Then the condition that the vev of the shifted field vanish in one-loop order fixes

$$\delta\bar{\zeta_v} = \frac{1}{16\pi^2}\left[3\lambda(1-\ln\frac{M^2}{\mu^2}) - 2N_c y^2 \frac{m^2}{M^2}(1-\ln\frac{m_t^2}{\mu^2})\right] \tag{32}$$

The field shift introduces V, and hence $\delta\zeta_v$, into the quadratic terms in (21) as well as generating the linear terms (26). This is the origin of the large electroweak contribution to the transformation from OS to $\overline{MS}$ for $M_h < m_t$.

In OS MOM renormalization, any of the Z's, as well as $\delta\zeta_v$. can contribute to finite part of (30). In addition to the condition of the vanishing of the vev of the shifted Higgs field, we have to specify a number of renormalization conditions equal to the number of bare parameters (μ_B^2, λ_B) and field multiplets (Φ_B) in the Lagrangian. We could take those to be the mass shell conditions (pole and residue)

$$\Sigma_\phi(0) = 0, \quad \Sigma_h'(M^2) = 0, \quad \Sigma_h(M^2) = 0. \tag{33}$$

(For $\overline{MS}$ these conditions have zero replaced by finite,i.e. $\overline{MS}$ counter terms remove all Δ_ϵ's) In our notations, the two-point counter terms appear in combinations

$$\begin{gathered} D_\phi^{-1}(q^2) = q^2 - \Sigma_\phi^{FD}(q^2) + (Z_\phi - 1)(q^2) + \delta\mu^2 \\ D_h^{-1}(q^2) = q^2 - M^2 - \Sigma_h^{FD}(q^2) + (Z_\phi - 1)q^2 + \delta\mu^2 - (Z_4\zeta_v^2 - 1)M^2 \end{gathered} \tag{34}$$

The same $\delta\mu^2$ (30) satisfies both $<\hat{h}>= 0$ and $\Sigma_\phi(0) = 0$. This is a manifestation of Goldstone's theorem and is a check on our renormalization procedure. From these conditions one can determine Z_ϕ, Z_{μ^2},and the product $Z_4\zeta_v^2$. For $\overline{MS}$, $\delta\bar{\zeta_v}$ is separately determined by the vanishing vev condition, so all the $\bar{Z}$'s are determined (and it is verified that $\delta\bar{\zeta_v}$ is finite). For OS MOM scheme another renormalization condition is required to separate Z_4 from ζ_v. For this, we can choose $h\phi_+\phi_-$ vertex. This is related to the above two-point functions by a Ward Identity.

$$V\Gamma_{h\phi_+\phi_-}(0,q) = D_\phi^{-1}(q^2) - D_h^{-1}(q^2) \tag{35}$$

Separate this into tree and one-loop and also write separately integrals from Feynman diagrams and counter terms.

$$\begin{gathered} (v+\delta V)(2\lambda v + \tilde{\Gamma}^{FD} + 2\lambda v(Z_4\zeta_v - 1)) = \\ q^2 - \Sigma_\phi^{FD} + (Z_\phi - 1)q^2 + \delta\mu^2 - q^2 + M^2 + \Sigma_h^{FD} - (Z_\phi - 1)q^2 - \delta\mu^2 + (Z_4\zeta_v^2 - 1)M^2 \end{gathered} \tag{36}$$

$$
\begin{aligned}
v 2\lambda v &= M^2 \\
v\tilde{\Gamma}^{FD}(0,q) &= -\Sigma_\phi^{FD}(q^2) + \Sigma_h^{FD}(q^2) \\
\delta V 2\lambda v + v 2\lambda v(\delta Z_4 + \delta\zeta_v) &= 0 + (\delta Z_4 + 2\delta\zeta_v) 2\lambda v^2
\end{aligned}
\tag{37}
$$

The last line is an identity, The Ward Identity is satisfied for any values of δZ_4 and $\delta\zeta_v$. We complete the the renormalization conditions for the OS MOM scheme by the condition

$$
v\Gamma_{h\phi_+\phi_-}(0,q)|_{q^2=M^2} = D_\phi^{-1}(M^2) \tag{38}
$$

This condition requires $\delta\zeta_v^\star$ to be zero ($V^\star = v^\star$) and Z_4 is now fixed by (33) with the Ward Identity then implying (38).

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Figure 1: One-loop tadpole diagrams with counter term

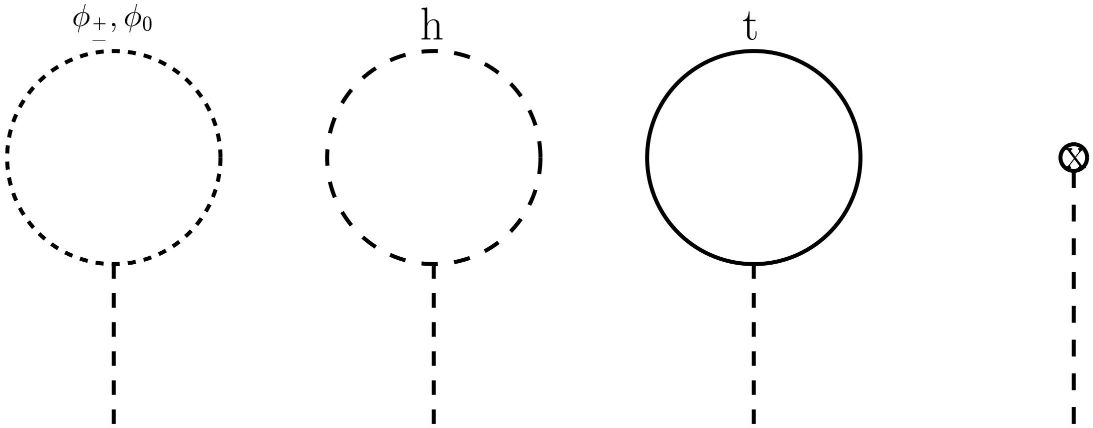

Figure 1

Table 1: t-quark $\overline{MS}$ mass $\bar{m}_t(\bar{m}_t)$

M_h	QCD(1-L)	QCD(2-L)	QCD+EW
100	166	164	185
150	166	164	174
200	166	164	170
250	166	164	168.6
300	166	164	168

Table 2: b-quark $\overline{MS}$ mass $\bar{m}_b(\bar{m}_b)$

M_h	QCD(1-L)	QCD(2-L)	QCD+EW
100	4.46	4.19	–
150	4.46	4.19	2.52
200	4.46	4.19	3.60
250	4.46	4.19	4.11
300	4.46	4.19	4.47

Table 3: b-quark $\overline{MS}$ mass at M_Z, $\bar{m}_b(M_Z)$

M_h	$\bar{m}$	t_0	F	$\bar{m}_b(M_Z)$
100	–			
150	2.52	-.665	.0528	2.27
200	3.60	-.308	.1679	2.84
250	4.11	-.196	.2133	3.17
300	4.47	-.0918	.2343	3.45

Table 4: b-quark $\overline{MS}$ mass $\bar{m}_b$ from sum rule

M_h	QCD	QCD+EW
100	4.46	–
150	4,46	2.86
200	4.46	3.63
250	4.46	3.98
300	4.46	4.19